International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 7, Issue 5, Aug - Sep 2018; 19-28

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

COMPARISON OF MULTIVARIATE EXPONENTIALLY WEIGHTED MOVING AVERAGE AND GENERALIZED VARIANCE |S|PROCEDURES WITH INDUSTRIAL APPLICATION

***M. S. Hamed*[1] *& S. M. Mohamed*[2]**

[1]*Department of Administrative Sciences, Faculty of Business Administration, Taibah University, KSA*

[1]*Department of Applied Statistics, Faculty of Commerce, Benha University, Egypt*

[2]*Department of Applied Statistics, Institute of Statistical Studies & Research, Cairo University, Egypt*

ABSTRACT

In this paper, comparing between two procedures: Multivariate Exponentially Weighted Moving Average (MEWMA) quality control chart and Generalized Variance |S| quality control chart. The first procedure MEWMA is an example of a multivariate charting scheme whose monitoring statistic is unable to determine which variable caused the signal. The second procedure is a Generalized Variance |S| quality control chart for the multivariate process, it is a very powerful way to detect small shifts in the mean vector. Generalized variance chart allows us to simultaneously monitor whether the joint variability of two or more related variables is in control. In addition, this paper provides a comparison between MEWMA and generalized variance |S| multivariate control chart procedures by application with real data.

KEYWORDS: *Average Run Length Performance, Monitoring Process, Multivariate Statistical Analysis, Quality Control*

2010 Mathematics Subject Classification (MSC) code: 62P30

Article History

Received: 04 Jul 2018* | *Revised: 28 Jul 2018* | *Accepted: 23 Aug 2018

1. INTRODUCTION

The drawbacks to multivariate charting schemes are their inability to identify which variable was the source of the signal. With today's use of computers, it is common to monitor several correlated quality characteristics simultaneously. Various types of multivariate control charts have been proposed to take advantage of the relationships among the variables being monitored. A control chart commonly requires samples with fixed size be taken at fixed intervals. It is assumed that in both univariate and multivariate control charts, each sample is independent of the previous samples.

2. COMPARISON METHODOLOGY

2.1 The MEWMA Control Chart

2.1.1The EWMA Control Chart Procedure

The EWMA techniques give the most recent observation the greatest weight with all previous observations weights decreasing in a geometric (exponential) progression from the recent back to the first. To demonstrate the EWMA technique, suppose that we observe sample means $X_1 . X_2 . X_3$, … in the univariate case, where,

$$X \sim N\left(\mu_0 . \sigma_x^2\right) \quad (1)$$

MacGregor (1995) as introduced the univariate Exponentially Weighed Moving Average (EWMA) control chart:

$$Z_i = rX_i + (1 - r)Z_{i-1} . i = 1.2.3. \quad (2)$$

where r is a smoothing constant and Z_i is the value of the EWMA after observation i. where irepresents the observation number as well as an index of a point in time, where we required $Z_0 = \mu_0 = 0$ without loss of generality and $0 < r \leq 1$. and he supposed that if $X_1 . X_2 . X_3$, … are independently and identically distributed $N(0.\sigma^2)$ random variables, the mean ofZ_i is0 and the variance is

$$\operatorname{var}(Z_i) = \sigma_Z^2 = \left\{\frac{r\left[1-(1-r)^{2i}\right]}{2-r}\right\} \sigma_X^2 . i = 1.2.3. \quad (3)$$

Thus, he suggested that when the in-control value of the mean is 0, the control limits of the EWMA chart are at $\pm L\sigma_{Zi}$. where L and r are the parameters of the chart.

2.1.2 Multivariate Extension of the EWMA Control Chart

Lowry et al. (1992) have generalized the concept of the univariate EWMA control chart to the multivariate case. They defined the MEWMA vectors as:

$$Z_i = RX_i + (1 - R)Z_{i-1} . i = 1.2.3. \quad (4)$$

where $Z_0 = 0$ and

$$R = \operatorname{diag}\left(r_1 . r_2 . \dots . r_p . 0 < r_j \leq 1\right) . j = 1.2. \dots . p \text{ and } p > 1.$$

The MEWMA control chart gives an out of control signal as soon as:

$$T_i^2 = Z_i^{`} \Sigma_i^{-1} Z_i > L \quad (5)$$

where $L > 0$ is chosen to achieve a specified in-control ARL, and Σ_{Zi} is the covariance matrix of Z_i.

Often there is no reason to apply different exponential weights to past observations of the p deferent quality characteristics. In this situation, Lowry et al. (1992) assumed the equal weights across characteristics where$r = r_j . j = 1. \dots$,p, the MEWMA vectors can then be written as

$$Z_i = rX_i + (1 - r)Z_{i-1} . i = 1.2.3. \dots \quad (6)$$

Under the assumption of equal weights, Lowry et al. (1992) have shown that the covariance matrix of Z_i can be written in terms of the exponential weight r and the covariance matrix of the process data $\Sigma_X$as:

$$\Sigma_{Zi} = \left\{\frac{r\left[1-(1-r)^{2i}\right]}{2-r}\right\} \Sigma_X \quad (7)$$

note that if $r = 1$, the MEWMA chart is equivalent to Hotellin's T^2 chart.

Lowry et al. (1992) have suggested that when the process is likely to stay in-control for some time period, the asymptotic form of the covariance matrix $\Sigma_X$used to calculate the MEWMA test statistic:

$$\Sigma_{Zi} = \left\{\frac{r}{2-r}\right\}\Sigma_X \tag{8}$$

They have depended on the use of MEWMA chart in order to study the *p* quality characteristics associated with a process. The process begins in the in-control state with mean vector $\mu_0 = 0$ and covariance matrix Σ_X. They supposed also that the process is subject to a single assignable cause, which shifts the process mean from μ_0 to a point on the constant probability density contour *D*; defined by

$$D = \left[\mu_1 \middle| \acute{\mu}_1 \Sigma_X^{-1} \mu_1 = \delta^2\right] \tag{9}$$

where δ, the parameter describing the size of the shift, is known. Note that Equation (9) is the constant probability density contour for a p- dimensional multivariate normal distribution. This contour forms an ellipsoid which is centered at μ_0 and has axes at

$$\pm \delta \sqrt{\xi_j e_j}, \text{ where} \Sigma_X e_j = \xi_j e_j . \text{ for } j = 1. \ldots, p.$$

2.1.3 The Average Run Length (ARL) Performance

The average run length is metric used to determine the control chart's ability in order to determine if the process is in control or out of control. The non-centrality parameter given by Lowry et al. (1995), as:

$$\delta^2 \left(\mu_y\right) = (\mu_y - \mu_0)' \Sigma_X^{-1} (\mu_y - \mu_0) \tag{10}$$

where μ_y is the mean when the process is out of control. They supposed that if $\mu_i = \mu$, $i = 1.2. \ldots$ the non-centrality parameter is given as:

$$\delta^2 = (\mu' \Sigma_X^{-1} \mu)$$

Recent researches have been developed to approximate the ARL for a MEWMA control chart. These methodologies have assumed that the in-control distribution is given as $N_p(\mu_0 . \Sigma_X)$.

2.2 Multivariate Process Variability Control Chart

Jackson (1961) proposed that the starting point of the statistical application of the method of principal components is the sample covariance matrix S. for a p-variate problem,

$$S = \begin{bmatrix} s_1^2 & s_{12} & \cdots & s_{1p} \\ s_{21} & s_2^2 & \cdots & s_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ s_{p1} & s_{p2} & \cdots & s_p^2 \end{bmatrix} \tag{11}$$

where s_i^2 is the variance of the i^{th} variable and s_{ij} is the covariance between the i^{th} and j^{th} variables.

The first procedure is a direct extension of the univariateS^2 control chart, where S^2 is the sample variance. The procedure is equivalent to repeated tests of significance of the hypothesis that the process covariance matrix is equal to the particular matrix of constants Σ. If this approach is used, the statistic plotted on the control chart for the i^{th} sample is

$$:W_i = -pn + pn \ln(n) - n \ln\left(\frac{A_i}{|\Sigma|}\right) + \operatorname{tr}(\Sigma^{-1} A_i). \tag{12}$$

where $A_i = (n-1)S_i$. S_i is the sample covariance matrix for sample i, and tr is the trace operator (the trace of matrix is the sum of the main diagonal elements) if the value of W_i plots above the upper control limit $UCL = \chi^2_{\alpha.\frac{p(p+1)}{2}}$, the process is out of control.

The |S| chart, as presented by Montgomery (2001, pp. 533-534) have dependent on the mean variance of |S|- that is,E(|S|) and V(|S|) - and the property that most of the probability distribution of |S| is contained in the interval: $E(|S|) \pm 3\sqrt{V(|S|)}$ as follows as:

$$\text{and} \left.\begin{array}{r} E(|S|) = b_1|\Sigma| \\ V(|S|) = b_2|\Sigma|^2 \end{array}\right\}. \tag{13}$$

where

$$b_1 = \frac{1}{(n-1)^p}\prod_{i=1}^{p}(n-i)$$

and

$$b_2 = \frac{1}{(n-1)^{2p}}\prod_{i=1}^{p}(n-i)\left[\prod_{j=1}^{p}(n-j+2) - \prod_{j=1}^{p}(n-j)\right]$$

Therefore, the parameters of the control chart for |S| would be

$$\left.\begin{array}{r} UCL = |\Sigma|(b_1 + 3\sqrt{b_2}) \\ CL = b_1|\Sigma| \\ LCL = |\Sigma|(b_1 - 3\sqrt{b_2}) \end{array}\right\} \tag{14}$$

Accordingly, we should replace $|\Sigma|$ in equation (14) by $\frac{|\Sigma|}{b_1}$. Since equation (13) has shown that $\frac{|S|}{b_1}$ is an unbiased estimator of$|\Sigma|$.

As a result, we find that the control chart parameters are

$$\left.\begin{array}{r} UCL = |S|(1 - \frac{3\sqrt{b_2}}{b_1}) \\ CL = |S| \\ LCL = |S|(1 - \frac{3\sqrt{b_2}}{b_1}) \end{array}\right\} \tag{15}$$

3. THE APPLICATION

Delta Fertilizers and Chemical Industries are considered one of the leading companies in the field of fertilizers production in Egypt. About 4500 employees are working for it, on the various managerial levels. Urea production is one of the major products of the company. For the application of multivariate quality control, chart data originate from the urea production process, which consists of the three stages and the analysis of laboratory, which discussed above. The number of the sample is 732 observations taken per hour. In this application,we shall introduce the most common using technique of multivariate quality control charts; MEWMA chart &generalized variance chart.

3.1 MEWMA Chart

A MEWMAchart consists of:

- Plotted points, each of which represents the multivariate statistic for each observation.

- Upper control limits (red), which provide a visual means for assessing whether the process is in-control.

We select a combination of r and ARL for plotting several MEWMA charts for each stage of the production study. Note that the default value of r = 0.5 and the default value of ARL= 100.

3.1.1. MEWMA Chart of X1,..., X16 and t1.1, …, t2.5

Test Results for MEWMA Chart of X1,..., X16 and t1.1, …, t2.5

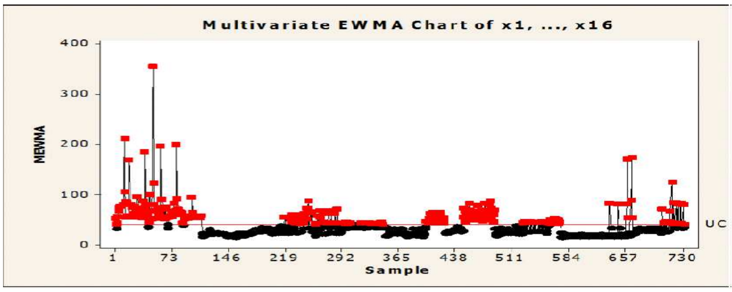

Figure 2: MEWMA Chart of X1,..., X16 and t1.1, …, t2.5

MEWMA Chart of X1,..., X16 and t1.1, …, t2.5 can be Summarized as Follows

- The upper control limit is 40.2. Therefore, we expect the MEWMA statistics to fall below 40.2.

- Test results indicate that 278 points beyond the control limits.

- Test results indicate that the process is in- control for 454 points and out –of control for 278 points. Then the out-of-control rate 37.98% and the in-control rate 62.02%.

3.1.2. MEWMAchart of y1, …, y7 and t3.1, …, t4.5

Test Results for MEWMAChart of y1, …, y7 and t3.1, …, t4.5

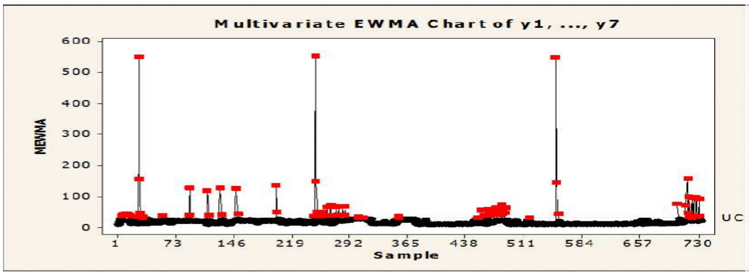

Figure3: MEWMA Chart of y1, …, y7 and t3.1, …, t4.5.

MEWMA Chart of y1, …, y7 and t3.1, …, t4.5 can be Summarized as Follows

- The upper control limit is 30.6. Therefore, we expect the MEWMA statistics to fall below 30.6.

- Test results indicate that 89 points beyond the control limits.

- Test results indicate that the process is in- control for 634 points and out –of control for 89 points. Then the out-of-control rate 12.16% and the in-control rate 87.84%.

3.1.3. MEWMA Chart of Z1, …, Z4 and t6.1, …, t6.8

Test Results for MEWMA Chart of Z1, …, Z4 and t6.1, …, t6.8

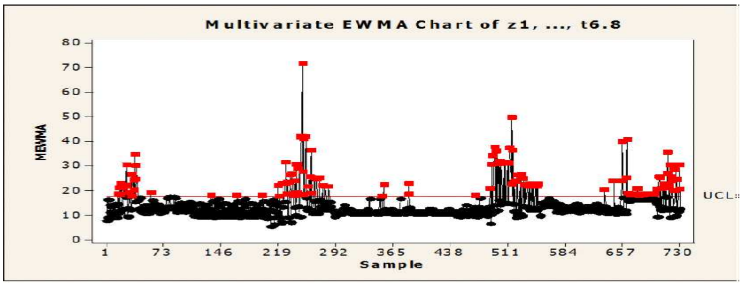

Figure 4: MEWMA chart of Z1, …, Z4 and t6.1, …, t6.8

MEWMA Chart of Z1, …, Z4 and t6.1, …, t6.8 can be Summarized as Follows:

- The upper control limit is 17.78 Therefore, we expect the MEWMA statistics to fall below 17.78.
- Test results indicate 135 points through beyond the control limits.
- Test results indicate that the process is in- control for 597 points and out –of control for 135 points. Then the out-of- control rate 18.44% and the in-control rate 81.56%.

3.2 Generalized Variance Chart

A generalized variance chart consists of:

- Plotted points, each of which represents the generalized variance for each observation.
- A center line (green), which is the median of the theoretical distribution of generalized variance statistic.
- Control limits (red), which provide a visual means for assessing whether the process is in-control. The control limits represent the expected variation.

MINITAB marks points outside of the control limits with a red symbol.

3.2.1. Generalized Variance Chart of X1,..., X16 and t1.1, …, t2.5

Test results for generalized variance chart of X1,..., X16 and t1.1, …, t2.5

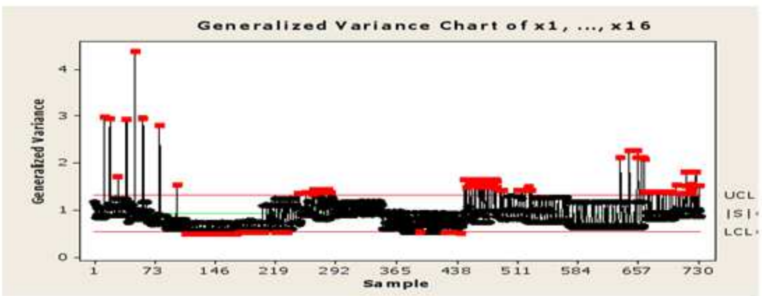

Figure 5: Generalized Variance Chart of X1,..., X16 and t1.1, …, t2.5

Generalized Variance Chart of X1,..., X16 and t1.1, …, t2.5 can be Summarized as Follows

- The lower and upper control limits are 0.528 and 1.312, respectively. Therefore, we expect the generalized variance statistics to fall between 0.528 and 1.312. The center line, or median, is 0.92.
- Test results indicate 111 points through beyond the control limits.
- Test results indicate that the process is in- control for 621 points and out -of control for 111 points. Then the out-of-control rate 15.16% and the in-control rate 84.84%.

3.2.2. Generalized Variance Chart of y1, …, y7 and t3.1, …, t4.5

Test results for generalized variance chart of y1, …, y7 and t3.1, …, t4.5

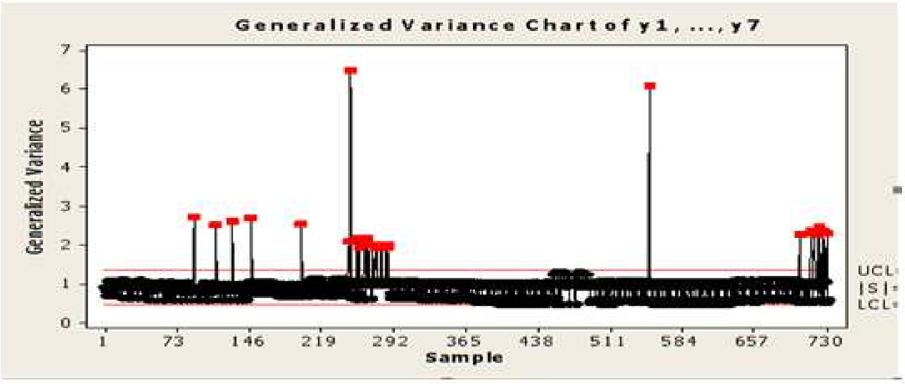

Figure 6: Generalized Variance Chart of y1, …, y7 and t3.1, …, t4.5

Generalized Variance Chart of y1, …, y7 and t3.1, …, t4.5 can be Summarized as Follows

- The lower and upper control limits are 0.456 and 1.330, respectively. Therefore, we expect the generalized variance statistics to fall between 0.456 and 1.330. The center line, or median, is 0.893.
- Test results indicate 31 points through beyond the control limits.
- Test results indicate that the process is in- control for 701 points and out -of control for 31 points. Then the out-of-control rate 4.23% and the in-control rate 95.77%.

3.2.3. Generalized variance chart of Z1, …, Z4 and t6.1, …, t6.8

Test results for generalized variance chart of Z1, …, Z4 and t6.1, …, t6.8

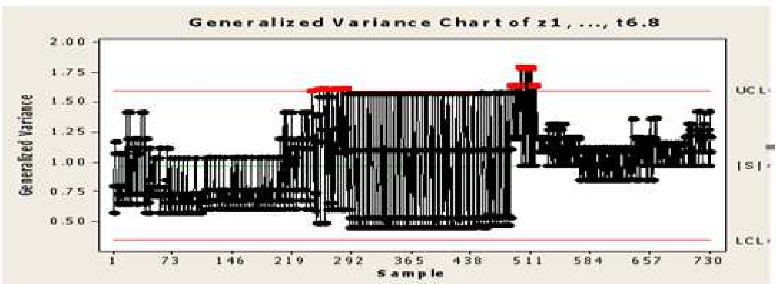

Figure 7: Generalized Variance Chart of Z1, …, Z4 and t6.1, …, t6.8

Generalized Variance Chart of Z1, …, Z4 and t6.1, …, t6.8 can be Summarized as Follows

- The lower and upper control limits are 0.342 and 1.594, respectively. Therefore, we expect the generalized variance statistics to fall between 0.342 and 1.594. The center line, or median, is 0.968.
- Test results indicate 26 points through beyond the control limits.
- Test results indicate that the process is in- control for 706 points and out -of control for 26 points. Then the out-of-control rate 3.55% and the in-control rate of 96.45%.

4. COMPARISON RESULTS OF APPLICATION

The application is shown that in High process stage, test results of MEWMA chart indicate that the out-of-control percentage 37.98% and the in-control percentage 62.02%, and it is shown that in Low process stage, test results of MEWMA chart indicate that the out-of-control percentage 12.16% and the in-control percentage 87.84%. It is shown that in the Evaporation and Prilling stage, test results of MEWMA chart indicates that the out-of-control percentage 18.44% and the in-control percentage 81.56%. While the application is shown that in generalized variance chart, in the High process stage, test results indicate that the out-of-control percentage 15.16%and the in-control percentage 84.84%, while in the Low process stage, the out-of-control percentage4.23% and the in-control percentage 95.77% and in the Evaporation and Prilling stage, the out-of-control percentage 3.55% and the in-control percentage 96.45%.

5. CONCLUSIONS

The results allow us to determine whether the joint process variability is in-control or out-of-control. It is shown that the out-of-control and in-control percentage changes by using difference values of r. It was shown that there is a relationship between the value of r and the out-of-control percentage, the out-of-control percentage increased by increasing the value of r and ARLH amed, M. S. (2016). The results are shown that in the design of MEWMA control charts, small values of r are more efficient in detecting small process mean shifts and large values of r are more efficient in detecting large process mean shifts. Generalized variance chart used to determine whether or not the joint process variability (the joint variability that accounts for the variability of each charted variable) for two or variables is in-control. Generalized variance charts allow us to simultaneously monitor whether the joint variability of two or more related variables is in-control.

Finally, On using the MEWMA chart to determine whether or not the process in control, the company should choose small values of r to detect small process mean shifts and choose large values of r to detect large process mean shifts.

- On using the generalized variance chart to determine whether or not the process in control, the company should choose the joint process variability for two or more variables is in control.

ACKNOWLEDGEMENT

The author is grateful to the associate editor and referees for careful reading and the Editor for very useful comments and suggestions.

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